arteche

SERIAL FERRORESONANCE IN VOLTAGE TRANSFORMERS

TRAINING BOOKLET: 6

Moving together

CONTENTS

SUMMARY

This article describes the serial ferroresonance phenomenon in the cases of fundamental and subharmonic ferroresonance. Starting from the complete R-L-C circuit in ferroresonance, the minimum network voltage which sustains the phenomenon is determined.

In the case of fundamental ferroresonance, the results of various laboratory experiments are compared with the carried out calculations. These points are immediately applicable to the case of inductive and capacitive voltage transformers.

1. DESCRIPTION OF THE SERIAL FERRORESONANCE PHENOMENON

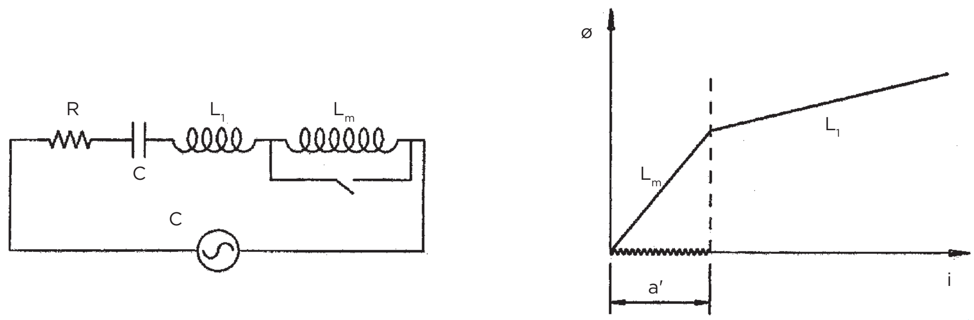
In a serial R-L-C circuit, for an applied voltage, a constant current always flows through the circuit, since its components are characterized by being constant and therefore non-variable with current, voltage or time.

However, if we introduce as a parameter of the same circuit a self inductance (1) variable with the current or the voltage in its terminals, we find ourselves with the possibility that a sustained range of currents and voltages may be produced within the circuit totally different from that expected for a determined supply voltage.

The classic method of explaining the phenomenon [1], although it gives sufficient initial approximation, doesn't lead to real understanding of the phenomenon and its mathematical quantification because it doesn't take into account that the self-induction is a complex transference curve and not the direct quotient between values of voltage and current in a continuous rating.

In fig. 1 we have the serial R-L-C circuit and the magnetic characteristic of self-inductance.

The appearance of the condition of ferroresonance in the circuit, starting from a permanent sine-wave condition, is always due to a variation in the supply voltage to the circuit, which may return to the original conditions but maintaining the ferroresonance.



› Fig. 1

Fig. 2 shows the voltage and current wave forms in the different elements of an R-L-C serial circuit. From this figure we can work out the following: a circuit in sustained ferroresonance there are 2 transient conditions every half-cycle of the network, and they are synchronized by the sine-wave current which circulates through the circuit.

The inductance goes sharply from the L_1 to the L_m value and vice versa, with two conditions of charge and discharge in opposite directions of C through it.

When the discharge and charge current in opposite directions of C through Lm reachs the value a', then $L = L_m$ becomes $L = L_1$. At this point the capacitance C discharges through L_1 with a transient current in semi-sine wave and frequency equal to:

$$f_o = \frac{1}{2\pi\sqrt{L_1 C}}$$

When the transient current of this discharge approaches to zero ($t = \pi/\omega_0$) then for current values lower than a', the self-inductance becomes $L = L_m$. The capacitor C charged in the opposite direction meets a very large L_m and an oscillation of frequency:

$$f_m = \frac{1}{2\pi\sqrt{L_m C}}$$

much lower than the one before is produced.

The current of this transient condition increases very slowly and hardly any voltage variation can be appreciated in the capacitance.

When the current of this slow transient added to the sinewave current of the network reaches the value a' then $L = L_1$ and the phenomenon repeats indefinitely.

The following must be taken into account:

When $L = L_m$ at the network frequency we find out that

$$L_m \omega >> \frac{1}{C\,\omega}$$

and therefore the circuit is very inductive. This implies that a weak sine-wave current of an inductive nature will circulate through the circuit in the intervals in which the selfinductance is not saturated, and will make nearly all the line voltage appear as rippled over the self-inductance voltage.

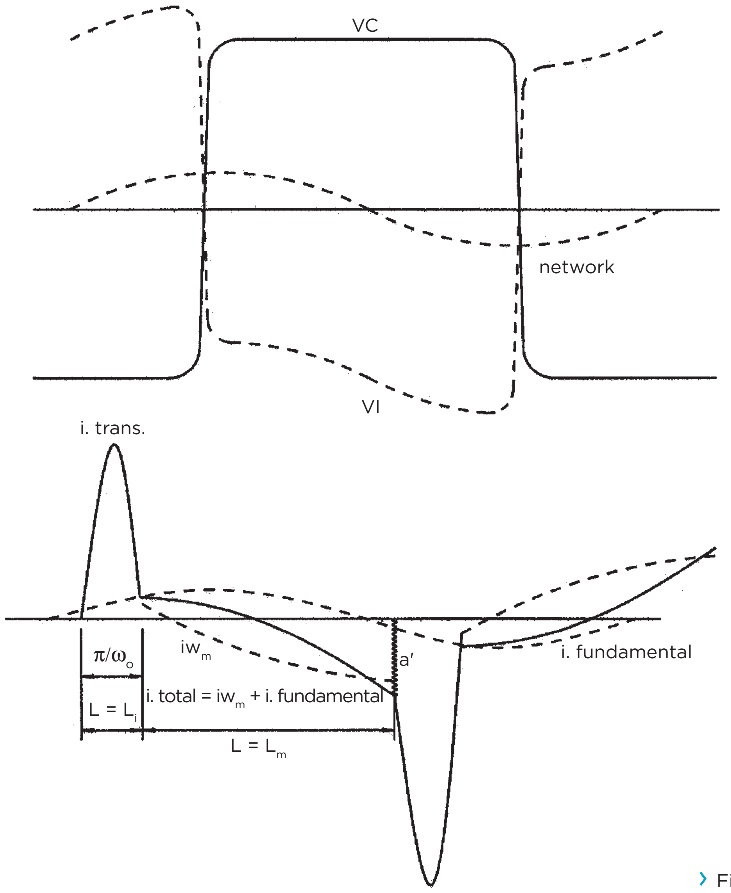


Fig. 2

2. FUNDAMENTAL AND SUBHARMONIC FERRORESONANCE

2.1. FUNDAMENTAL

From what has been described in the section above, we can work out the conditions which a R-C-L circuit must fulfill in order to maintain a fundamental ferroresonance:

a. The characteristic oscillation frequency of the L_1-C circuit must be higher than that of the network (usually much higher), that is to say:

$$\frac{1}{\sqrt{L_1 \cdot C}} \cong \omega_o > \omega$$

This is the same as saying that at the network frequency, the L_1-C circuit should be capacitive, that's to say:

$$\frac{1}{\omega \cdot C} > L_1 \omega$$

b. The characteristic oscillation frequency of the L_m-C circuit must be lower than that of the network, since in this way the phenomenon is synchronized every half-cycle. If $\omega_m > \omega$ then it is impossible to synchronize it. That is:

$$\frac{1}{\sqrt{L_m C}} = \omega_m < \omega$$

This is equivalent to saying that at the network frequency, the L_m-C circuit must be inductive and therefore:

$$\frac{1}{C \omega} < L_m \omega$$

c. The discharge of C through L_1 must be in an oscillating way, for which the circuit must be under-damped, that is:

$$R < R_c = 2\sqrt{\frac{L_1}{C}}$$

In this way we guarantee that the current passes through zero, with a $\omega_o > \omega$, and L_1 is converted to the value of L_m and vice versa.

The network voltage must be enough to provide the energy lost in the resistor R while in communication In addition we assume that the losses in R because of the sine-wave current of the network or the pulse current ω_m are neglected due to their low values.

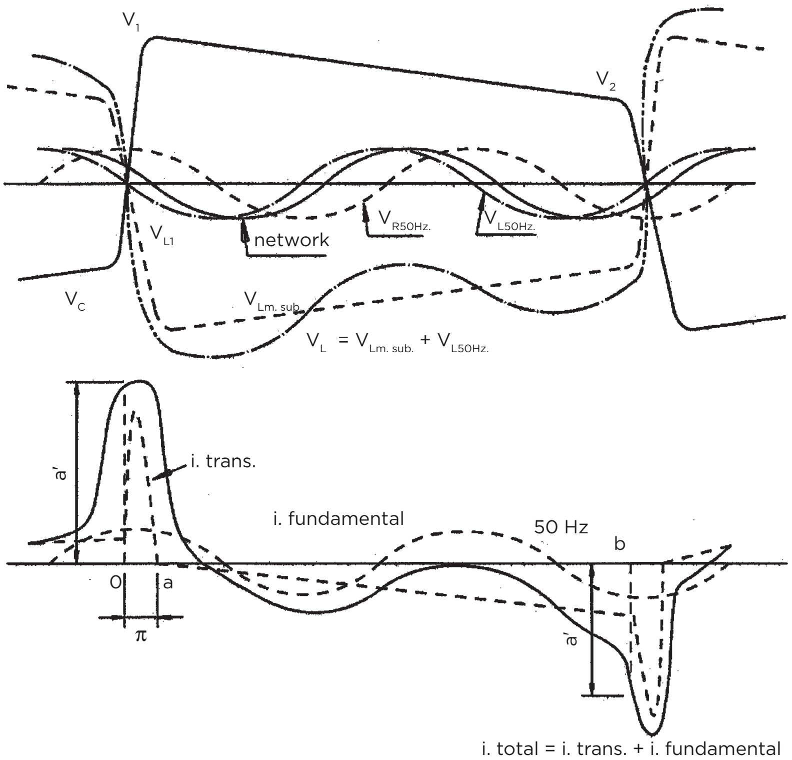
2.2. SUBHARMONIC

In fig. 3 we can see the wave shapes of voltages and currents in the circuit in the case of subharmonic ferroresonance.

The subharmonic ferroresonance is produced when the nonsaturated self-inductance L_m together with the capacitance C of the circuit have their own oscillation frequency of f_m, lower than that of the subharmonic which may be produced.

In this way, the oscillating current of frequency fm and the fundamental component added together may be able to saturate the self-inductance synchonicity every 1.5 network cycles, 2.5 cycles, 3.5 cycles, etc., that's to say, with an oscillation period of 3 cycles, 5 cycles, 7 cycles, etc. (3rd subharmonic,
5th subharmonic, 7th subharmonic, etc.).

The case above on fundamental ferroresonance is in fact the same, but synchronized every 1/2 cycle.



› Fig. 3

2. FUNDAMENTAL AND SUBHARMONIC FERRORESONANCE

Figure 3 shows that if the free oscillation frequency

$$\frac{1}{\sqrt{L_m C}}$$

is, for example, lower than the 9th subharmonic, ferroresonances of the 1st subharmonic (fundamental) may appear, then 3rd subharmonic and 5th subharmonic ones, followed by the 7th and even the 9th ones, but never the 11th, that is, at frequencies lower than that of free oscillation:

$$\frac{1}{\sqrt{L_m C}}$$

Subharmonics increase their periods (orders) while network voltage decreases.

From figure 3 we can deduce the circuit conditions required for the existence of subharmonic ferroresonance:

a. The characteristic oscillation frequency of the circuit L_1-C must be higher than that of the considered subharmonic, i.e.:

$$\omega_0 = \frac{1}{\sqrt{L_1 C}} > \omega_{subharmonic}$$

or just at the frequency of the considered subharmonic the circuit L_1-C must be capacitive:

$$\frac{1}{C\frac{\omega}{n}} > L_1 \frac{\omega}{n}$$

where n = order of the considered subharmonic.

b. The characteristic oscillation frequency of the circuit L_m-C must be lower than that of the considered subharmonic so that the phenomenon can be synchronized, i.e.:

$$\frac{1}{\sqrt{L_m C}} < \omega_{considered\ subharmonic}$$

This is equivalent to saying that at the frequency of the considered subharmonic the circuit L_m-C must be inductive:

$$\frac{1}{C\frac{\omega}{n}} > L_m \frac{\omega}{n}$$

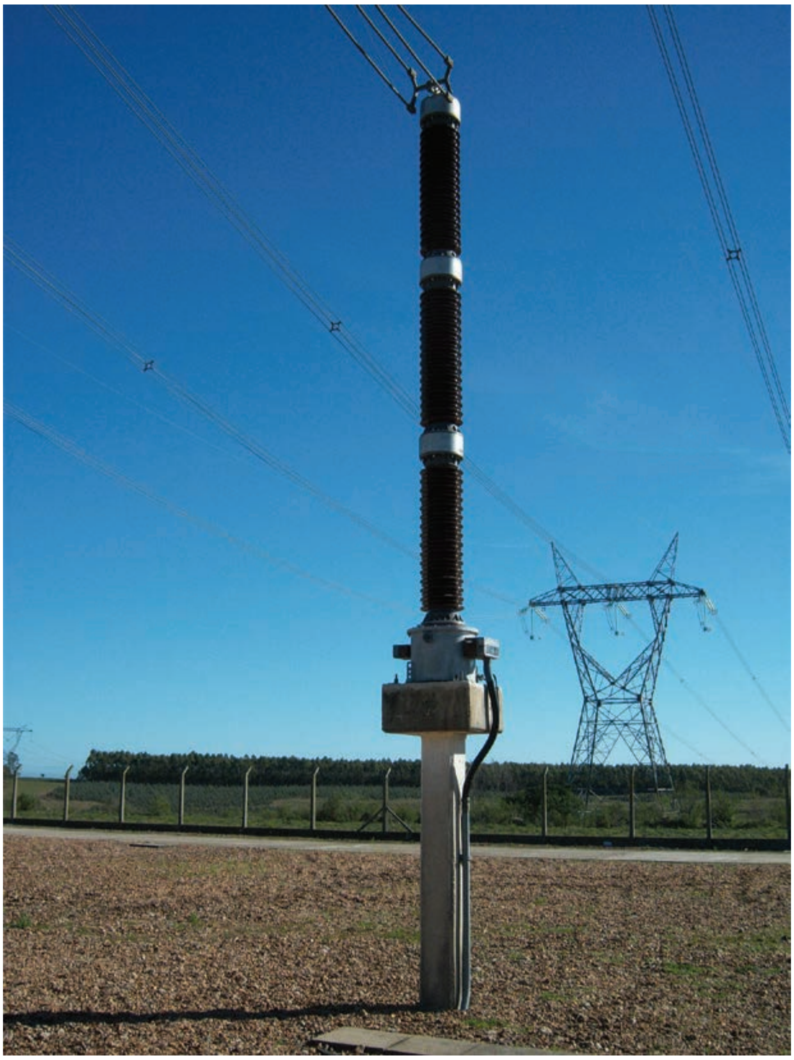
where n = order of the considered subharmonic.

c. The discharge of C through L_1 must be oscillating, for which the circuit must be underdamped, that is to say:

$$R < R_c = 2\sqrt{\frac{L_1}{C}}$$

In this way, we guarantee that the current passes through zero, with a $\omega_0 > \omega_{subharmonic}$, and L_1 is converted to the value L_m and vice versa.

d. The network voltage must be enough to provide the energy lost in the resistance R while commutation. In addition we assume that the losses in R because of the sinewave current of the network or the pulse current $\frac{\omega}{n}$ are neglected due to their low values.



3. CALCULATIONS AND EQUATIONS

The equilibrium equations for the maintenance of the fundamental and the subharmonic ferroresonance are shown below.

Primarily, we assume that the core has no losses and we check the influence of the serial resistance r_1. (fig.4).

The initial hypothesis are as follows:

a. The excitation sine-wave current is insignificant compared to the impulses while the transients.

b. Self-inductance has 2 values: L_m when it is not saturated, and L_1 when it is saturated. Also when the self-inductance value is L_m, it is so large that it prevents the capacitors from discharging.

The equations to be applied are:

$$U_{alimentation} = V_L + V_c \qquad \text{Eq. (1)}$$

Energy supplied by the network = Energy consummed inthe circuit Eq. (2)

Developing both equations for the case of fundamental ferroresonance and integrating equation (1) between a and b we have:

$$\int_a^b U_{alim} \cdot d(\omega \cdot t) = \int_a^b V_L \cdot d(\omega \cdot t) + \int_a^b V_C \cdot d(\omega \cdot t)$$

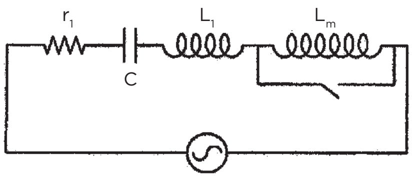
and where:

$$\int_a^b V_L \cdot d(\omega \cdot t) = \frac{E_{sat} \cdot 2 \cdot \sqrt{2}}{\omega}$$

$$\int_a^b V_C \cdot d(\omega \cdot t) = V_0\,(b - a)$$

$$b - a = \frac{\pi}{\omega} - \frac{\pi}{\omega_0}$$

given $m = \dfrac{\omega}{\omega_0}$ y $\omega_0 = \sqrt{\dfrac{1}{L_1 C} - \dfrac{r_1^2}{4L_1^2}}$



we have:

$$E_{sat} \cdot 2\sqrt{2} = \omega \cdot V_0\,(b - a)$$

$$E_{sat} \cdot 2\sqrt{2} = \omega \cdot V_0 \left(\frac{\pi}{\omega} - \frac{\pi}{\omega_0}\right)$$

$$E_{sat} \cdot 2\sqrt{2} = V_0\,\pi \left(1 - \frac{\omega}{\omega_0}\right)$$

$$E_{sat} \cdot 2\sqrt{2} = V_0\,\pi\,(1 - m)$$

Eq. (3)

From equation (2) we deduce:

Energy supplied by the network:

$$\int_{b'}^{a} u_{alim} \cdot i \cdot dt$$

Ec. (4)

Energy lost while commutation:

$$\int_{b'}^{a} r_1 \cdot i^2 \cdot dt$$

Eq. (5)

While commutation we can assume:

$$u_{alim} = \hat{V} = \sqrt{2} \cdot U_{eff}$$

since $\omega_0 >> \omega$

Similarly,

$$i = \hat{I} \cdot \text{sen}\ \omega_0 t$$

with $b' = 0$ and $a = \pi$

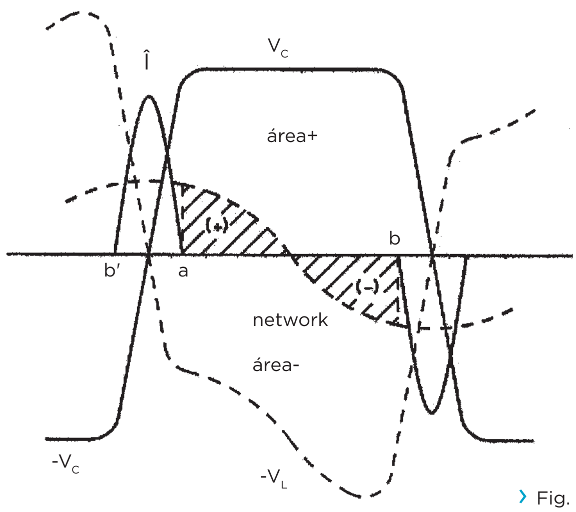


Fig. 4

3. CALCULATIONS AND EQUATIONS

Charging and discharging current of C implies a charge which value is:

$$\int_0^{\frac{\pi}{\omega_0}} i dt = 2 \cdot C \cdot V_0$$

Supposing that $i = \hat{I} \cdot \text{sen}\ \omega_0 t$

$$\Rightarrow \int_0^{\frac{\pi}{\omega_0}} \hat{I} \cdot \text{sen}\ \omega_0\ t dt = \hat{I} \cdot \frac{2}{\omega_0}$$

That's to say: $\hat{I} \cdot \frac{2}{\omega_0} = 2 \cdot C \cdot V_0$

As: $\omega = m \cdot \omega_0$

$$\Rightarrow \hat{I} = \frac{\omega}{m} \cdot C \cdot V_0$$

Therefore:

$$\hat{I} = \frac{V_0}{m \cdot \frac{1}{\omega \cdot C}} = \frac{V_0}{m \cdot X_c}$$

The relation between the minimum voltage which ($\hat{V}$), maintains the phenomenon, the voltage in the capacitor (V_0) and the current while conmutation periods ($\hat{I}$) is obtained:

$$\hat{V} \int_0^{\frac{\pi}{\omega_0}} i \cdot dt = r_1 \int_0^{\frac{\pi}{\omega_0}} i^2 \cdot dt$$

as $i = \hat{I} \cdot \text{sen}\ \omega_0 t$, we obtain

$$r_1 \cdot \int_0^{\frac{\pi}{\omega_0}} \hat{I}^2\ \text{sen}^2\ \omega_0 t dt = r_1 \hat{I}^2 \int_0^{\frac{\pi}{\omega_0}} \left(\frac{1 - \cos 2\omega_0 t}{2} \right) dt$$

$$= r_1 \hat{I}^2 \left[\frac{t}{2} - \frac{1}{4\omega}\ \text{sen}\ 2\omega t \right]_0^{\frac{\pi}{\omega_0}}$$

$$r_1 \hat{I}^2 \left[\frac{\pi}{2\omega_0} - 0 \right] = \frac{r_1 \hat{I}^2 \pi}{2\omega_0}$$

From another way:

$$\int_0^{\frac{\pi}{\omega_0}} i \cdot dt = Q = 2 \cdot C \cdot V_0$$

Supposing that $\hat{V}$ remains constant along that semiperiod we have:

$$\hat{V} \int_0^{\frac{\pi}{\omega_0}} i \cdot dt = \hat{V} \cdot Q = \hat{V} \cdot 2 \cdot C \cdot V_0$$

Equalling both terms:

$$\hat{V} \cdot 2 \cdot C \cdot V_0 = \frac{r_1 \hat{I}^2 \pi}{2\ \omega_0}$$

As

$$2 \cdot C \cdot V_0 = \frac{\hat{I} \cdot 2}{\omega_0}$$

we have

$$\hat{V} \cdot \frac{\hat{I} \cdot 2}{\omega_0} = \frac{r_1 \cdot \hat{I}^2 \cdot \pi}{2 \cdot \omega_0}$$

that's to say:

$$\hat{V} = \frac{r_1 \cdot \hat{I} \cdot \pi}{4}$$

As $\hat{I} = \frac{\omega}{m} \cdot C \cdot V_0$ y $V_0 = \frac{E_{sat} \cdot 2\sqrt{2}}{\pi \cdot (1-m)}$

we have:

$$\hat{V} = \frac{r_1 \cdot \omega \cdot C \cdot E_{sat} \cdot 2\sqrt{2} \cdot \pi}{m \cdot 4 \cdot \pi \cdot (1-m)}$$

that's to say:

$$\hat{V} = \frac{r_1 \cdot \omega \cdot C \cdot E_{sat} \cdot \sqrt{2}}{2m \cdot (1-m)} = \frac{r_1 \cdot E_{sat} \cdot \sqrt{2}}{2 \cdot X_c \cdot m \cdot (1-m)}$$

As $\hat{V} = U_{eff} . \sqrt{2}$, then:

$$\frac{U_{eff}}{E_{sat}} = \frac{r_1}{2 \cdot X_c \cdot m \cdot (1-m)}$$

Ec. (7)

where $m = \frac{\omega}{\omega_0}$, $X_c = \frac{1}{\omega \cdot C}$

E_{sat} = saturation voltage of the self-inductance at 50 Hz.

3. CALCULATIONS AND EQUATIONS

The influence of the losses in the self-inductance core can be represented by means of a resistor R in parallel with the non-saturated self-inductance L_m (fig.5).

The equation which has to be fulfilled with at any ferroresonance condition is as follows:

Energy supplied by the network = Energy lost in r_1 + Energy lost in R

Eq.(8)

Equation 8 is developed in the same way as the previous cases, with the only difference that the term "Energy lost in R" is obtained as the discharge from C through R when the self-inductance value is $L = L_m$.

Since the voltage in the capacitor falls from V_1 to V_2 we have a lost energy of value equal to:

$$\frac{1}{2} \cdot C \cdot (V_1^2 - V_2^2)$$

Besides in the charge-discharge process of C from V_2 to V_1 we have:

$$\int_0^{\frac{\pi}{\omega_0}} i \cdot dt = Q = (V_1 + V_2) \cdot C$$

and this is equal to:

$$\frac{\hat{I} \cdot 2}{\omega_0} = C \cdot (V_1 + V_2)$$

Therefore the feeding voltage increase $\Delta\hat{U}$ needed to maintain the ferroresonance will be:

$$\Delta\hat{U} \cdot \frac{\hat{I} \cdot 2}{\omega_0} = \Delta\hat{U} \cdot C \cdot (V_1 + V_2) = \frac{1}{2} \cdot C \cdot (V_1^2 - V_2^2)$$

That's to say:

$$\Delta\hat{U} = \frac{1}{2} \cdot (V_1 - V_2)$$

Or

$$\Delta\hat{U}_{eff} = \frac{V_1 - V_2}{2\sqrt{2}}$$

being

$$V_1 - V_2 = \frac{2\sqrt{2} \cdot E_{sat}}{\omega \cdot C \cdot R}$$

so

$$\frac{\Delta\hat{U}_{eff}}{E_{sat}} = \frac{1}{\omega \cdot C \cdot R} = \frac{X_c}{R}$$

Ec. (9)

Therefore, if we want to take into account the losses in the self-inductance, we have to add to the second term of equation (7) the second term of equation (9).

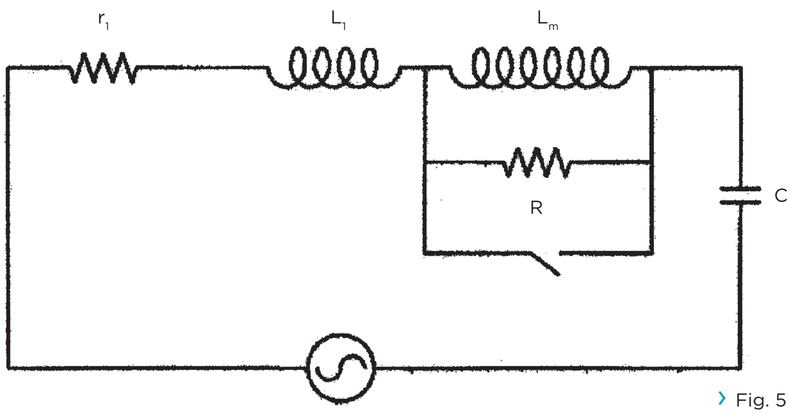


Fig. 5

4. RESULTS OF TESTS ON CIRCUITS IN FUNDAMENTAL FERRORESONANCE

The following experiments have been carried out on the circuit shown in figure 5, making it go into ferroresonance by raising the supply voltage. Afterwards the voltage supply is slowly reduced and the effective voltage at which the phenomenon disappears is measured. The obtained results appear in table 1.

Table 1

Circuit Parameters				Feeding Voltage	
r_1 Ohm	L_1x10^{-3}H	R Ohm	Cx10^{-6}F	CALCULATED	MEASURED
2.85	6.54	230	80	12.68	10.6
2.85	6.54	230	180	14.56	14.05
2.85	6.54	230	240	19.96	16.5
2.85	6.54	230	300	19.81	19.8
2.85	6.54	230	360	23.08	23.7
2.85	6.54	66	80	33.06	32
2.85	6.54	66	180	23.62	24.2
2.85	6.54	66	240	23.75	24.5
2.85	6.54	66	300	25.24	25.6
2.85	6.54	66	360	27.6	27.4

5. REFERENCES

[1] Cahen, F.: Electrotechnique, Gauthier-Villars, 1963.
[2] Mahy, P.: Contribution theorique et experimentale à l'etude des phènomènes de ferrorresonance monophasée, SRBE, 1972.

arteche
Moving together

ARTECHE_CF_Ferro_EN
Version: A0